

High-colour pomerons with a running coupling constant in the Hartree-Fock approximation

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Abstract. The Hartree-Fock approximation is applied to study the "high-colour pomerons" in the system of many reggeized gluons with a running QCD coupling constant. It is shown that, contrary to the fixed coupling case, the high-colour pomerons result supercritical, although with a smaller intercept than the multipomeron states.

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1. Introduction. We have recently proposed a new method to incorporate the running QCD coupling constant into the equation for two reggeized gluons, based on the so-called "bootstrap condition" which requires that in the gluon colour channel the solution should be the reggeized gluon itself [1]. As a result, the gluon Regge trajectory ω and its interaction U are expressed via a single function η . In the momentum space

$$\omega(q) = -(3/2)\eta(q) \int (d^2q_1/(2\pi)^2)/\eta(q_1)\eta(q_2) \quad (1)$$

and

$$U(q, q_1, q'_1) = -3(\eta(q_1)/\eta(q'_1) + \eta(q_2)/\eta(q'_2))/\eta(q_1 - q'_1) + 3\eta(q)/\eta(q'_1)\eta(q'_2) \quad (2)$$

In (2) q is the total momentum of the two interacting gluons. In both equations $q_1 + q_2 = q$. The fixed coupling constant BFKL pomeron [2] corresponds to the choice $\eta(q) = (2\pi/g^2)q^2$. The running coupling constant requires that $\eta(q) \simeq (2\pi/g^2(q))q^2$ for large q . Parametrizing the confinement effects by a "gluon mass" m we chose in [1]

$$\eta(q) = (b/2\pi)(q^2 + m^2) \ln((q^2 + m^2)/\Lambda^2) \quad (3)$$

where $b = (11 - (2/3)N_F)/4$ and $\Lambda \simeq 200 \text{ MeV}$ is the standard QCD parameter. Variational calculations with (1)-(3) gave the pomeron intercept weakly dependent on m and its slope rapidly falling with m [1]. With the slope of the order 0.2 GeV^{-2} , as favoured by the experimental data, the gluon mass results in the region $0.6 \div 0.8 \text{ GeV}$ and the intercept is of the order 0.35.

Thus nothing very spectacular occurs for the perturbative pomeron upon the introduction of the running coupling, except, of course, a nontrivial slope, which, after all, has to be expected, since the BFKL pomeron does not contain parameters with a dimension (see [3]). In this note we study the effect of the running coupling constant on the spectrum of the system of many reggeized gluons. This system is described by the Bartels-Kwiecinsky-Praszalowicz equation [4], which is nothing but a Schroedinger equation for many gluons in the transverse space in which the gluon trajectory serves as a kinetic energy (with a minus sign) and U provides for the interaction. Exact solutions of this equation are not known for the number of gluons $n \geq 2$. However there is an intense activity around this problem in the limit of large number of colours N_c , when, with a fixed coupling constant, the system turns out to be integrable [5,6]. For the realistic case $N_c = 3$ and fixed coupling, the odderon case ($n=3$) was studied in the variational approach in [7] with the result that the odderon has an intercept greater than 1 although smaller than the pomeron. Also for fixed coupling we studied the n -gluon problem in the Hartree-Fock approximation (HFA) [8]. Looking for solutions different from the multipomeronic ones we tried to evaluate the intercept of states symmetric both in colour and ordinary space variables. The result proved to be disappointing:

in the HFA such states possess intercepts lower than unity and going to the left linearly in n . Such states cannot contribute at high energies.

The purpose of this note is to report that the introduction of the running coupling changes the picture radically. Now the "energy" per gluon results negative, which means that the intercepts of symmetric multigluon states are above unity and go the right linearly with n . They still stay lower than those of the multipomeronic states which also exist in the n -gluon system. However at high energies the symmetric multigluon states ("high-coloured pomerons") do not die out and give a nontrivial factor to the standard contribution coming from multipomeron states. We recall that such high-coloured pomerons were introduced as high coloured strings in a more phenomenological treatment in [9]. They seem to lead to a better agreement with experiment for strange baryon production [10] and could be felt in long-order correlations [11]. The results of the present study give some theoretical foundation for the existence of high-coloured strings.

2. The HFA for reggeized gluons. After the change of the wave function $\psi \rightarrow \prod \eta_i \psi$ where $\eta_i = \eta(q_i)$, the equation for n reggeized gluons acquires the form

$$H\psi = E \prod_{i=1}^n \eta_i^2 \psi \quad (4)$$

The Hamiltonian H is given by a sum of pair terms

$$H = -(1/6) \sum_{i < k} T_i T_k H_{ik} \quad (5)$$

Here T_i is the colour vector of the i th gluon. In a colourless state $\sum_{i=1}^n T_i = 0$. The pair Hamiltonian H_{ik} is

$$H_{ik} = \prod_{j=1, j \neq i, k}^n \eta_j (-\eta_i \eta_k (\omega_i + \omega_k) + \eta_i V_{ik} \eta_k + \eta_k V_{ik} \eta_i + Q_{ik}) \quad (6)$$

where $\omega_i = \omega(q_i)$; V_{ik} is a local potential acting between the gluons i and k with the Fourier transform $\eta(q)$:

$$V_{ik} = V(r_{ik}) = \int (d^2 q / 4\pi^2) \exp(iqr_{ik}) / \eta(q) \quad (7)$$

and Q_{ik} is a separable interaction whose action on the wave function is defined as

$$Q_{ik} \psi(\dots q_i, \dots q_k, \dots) = \eta(q_i + q_k) \int (d^2 q / 4\pi^2) \psi(\dots q'_i, \dots q'_k, \dots) \quad (8)$$

with $q_i + q_k = q'_i + q'_k$ and $q'_i - q'_k = 2q$. The intercept is given by the energy with a minus sign

$$E = 1 - j = -\Delta \quad (9)$$

so that the rightmost singularity in the complex angular momentum plane corresponds to the ground state. The solution of (4) may be found by a variational approach, as realizing the minimum value of the functional

$$\Phi = \int \prod_{i=1}^n d^2 r_i \psi^* H \psi \quad (10)$$

with the normalization condition

$$\int \psi^* \prod_{i=1}^n d^2 r_i \eta_i \psi = 1 \quad (11)$$

Eq. (4) mixes the colour and space variables. So its solution cannot be generally presented as a product of a colour wave function and a spatial one, both symmetric in their respective variables, except in some special cases ($n=2,3$ and the limit $N_c \rightarrow \infty$). In the HFA, however, one assumes that each gluon is moving in some average field, which in a colourless state cannot depend on colour. So in the HFA the colour and spatial variables decouple and the total wave function may be taken as a product of two symmetric functions in colour and in space. This allows to simplify the variational treatment considerably. Indeed, in a colourless state with a symmetric colour wave function

$$\langle T_i T_k \rangle = (2/n(n-1)) \sum_{i < k} T_i T_k = -3/(n-1) \quad (12)$$

where $T^2 = 3$ has been used. With this result and the symmetry of the spatial wave function we find that the energy can be found from the minimum of a simpler functional in only spatial variables

$$\mathcal{E} = (1/2) \int \prod_{i=1}^n d^2 r_i \psi^* H_{12} \psi \quad (13)$$

where the Hamiltonian H_{12} is defined by (6) with $ik = 12$ and the function ψ should satisfy (11). The energy of the whole system of n gluons is determined by the minimal value ϵ of \mathcal{E} according to

$$E_n = (1/2) n \epsilon_n \quad (14)$$

Actually the operator in \mathcal{E} does not depend on n . The dependence on n enters only from extra arguments in ψ through the requirement of the symmetry in all arguments.

In the HFA one takes for ψ a product

$$\psi = \prod_{i=1}^n \psi(r_i) \quad (15)$$

with the normalization

$$\int d^2 r \psi^* \eta \psi = 1 \quad (16)$$

Then \mathcal{E} reduces to a functional which involves the gluons 1 and 2 exclusively and does not depend on n altogether:

$$\mathcal{E} = (1/2) \int d^2r_1 d^2r_2 \psi^*(r_1) \psi^*(r_2) \tilde{H}_{12} \psi(r_1) \psi(r_2) \quad (17)$$

where \tilde{H}_{12} is given by (6) without the first factor (the product of all η_i except $i = 1, 2$). As a result in the HFA ϵ does not depend on n and the total energy is linear in the number of gluons

$$E_n^{HF} = (1/2)n\epsilon \quad (18)$$

If $\epsilon < 0$ all high-coloured pomerons result supercritical in this approximation, the intercept growing linearly with n . If $\epsilon > 0$ they are all subcritical and the intercept moves to the left linearly in n .

Calculations carried out in [1] for a fixed coupling constant gave $\epsilon \simeq (3\alpha_s/\pi)0.959$, so that all high-coloured states turned out subcritical and irrelevant at high-energies (in the HFA).

3. Calculations with the running coupling constant. With the running coupling we choose $\eta(q)$ according to (3) with the gluon mass m as a parameter. From the comparison of the pomeron slope to experimental data its value can be taken to lie in the range $0.6 \div 0.8 \text{ GeV}$. The functional \mathcal{E} was studied on the gluonic functions $\psi(r^2)$ which were taken to be a sum of Gaussians

$$\psi(r^2) = \sum_{k=1}^N c_k \exp(-\alpha_k r^2/2) \quad (19)$$

with $\alpha_{k+1} = c\alpha_k$. The calculation is straightforward, although very time consuming due to a large number of successive integrations (up to 6) and the necessity of a high precision because of cancellations between the kinetic and potential energies. The energy seems to go down with the rise of α_N but the calculational error rapidly grows with it. For that reason we had to limit ourselves with $N = 3$ and have chosen $c = 3$. In the end the variational parameters are α_1 and c_k .

The results of the calculations for m in the region $0.28 \div 2.0 \text{ GeV}$ are presented in the Table. Intercepts per gluon $\delta = \Delta/n$ together with the optimal values for α_1 and c_k are given in the Table. To see the convergence in N the results for $N = 2, 3$ are shown. One observes that δ is negative for values of m close to the value of $\Lambda = 0.2 \text{ GeV}$ in agreement with the fact that in the limit $m \rightarrow \Lambda$ our equation goes over into the BFKL one [1]. However with the rise of m the intercept goes up rather steeply and becomes positive already at $m = 0.4 \text{ GeV}$. It then stays practically independent of m , of the order $\delta = 0.08 - 0.09$, in the region $m = 0.6 \div 2.0 \text{ GeV}$ slightly falling at the end. With the realistic values of m

of the order $0.6 - 0.8 \text{ GeV}$, this means that the high-coloured pomerons are supercritical and their intercept goes up linearly with n . To compare, we present the intercepts per gluon δ_P of the $n/2$ pomeron state which is also present in the multigluon system and corresponds to a multipomeron cut in the old Regge-Pomeron theory (one half of the pomeron intercept calculated in [1] for different values of m). We observe that the intercepts of the high-colour pomerons are on the average nearly twice smaller than those of the mutipomeronic states in the studied range of m .

4. Discussion. Although the obtained results are rather direct and transparent by themselves, they should be taken with some caution. First, the fact that the "energy" of the new high-colour pomeron lies above the energy of multipomeron states means that it is unstable and decays into pomerons with less colour, eventually into ordinary pomerons. It is difficult to estimate its lifetime. It may be not so small because of the difference in the space structure of the new pomeron, with a wave function symmetric in all gluons and rapidly falling at large distances, and a mutipomeronic state, which is not symmetric in gluons and has a wave fucnction which does not fall as the pomerons go apart. Still the decays may essentially change the role of the new states at high energies.

Another suspicious point is that the parameter α_1 in the optimal wave function turns out to be of the same order as the gluon mass squared. This parameter, in fact, determines the average momentum squared. Thus the averaged momenta of our solutions are of the order m and lie in the nonperturbative region. In other words, our new pomerons have a transverse dimension comparable to that of a hadron. That means that they should be sensitive to what we assume about the behaviour of $\eta(q)$ at low q . Negative ϵ 's we have obtained may be a result of our particular choice of $\eta(q)$ and may turn into positive ones for a different choice (say, $\eta(q) = (2\pi/b)q^2 \ln((q^2 + m^2)/\Lambda^2)$). This does not make our results invalid, since, after all, the behaviour of $\eta(q)$ at small q is completely unknown and out of control. An experimental confirmation of the existence of high-coloured strings would be a decisive argument in this respect.

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Table. High-colour pomeron intercepts per gluon

m	$\delta(N = 2)$	$\delta(N = 3)$	δ_P	α_1	$c_k(N = 3)$
0.28	-0.0281	-0.0212	0.290	0.0590	0.957,-0.107,0.269
0.4	0.0427	0.0486	0.225	0.176	0.940,-0.146,0.309
0.6	0.0752	0.0820	0.186	0.324	0.910,-0.208,0.358
0.8	0.0816	0.0896	0.168	0.544	0.888,-0.358,0.381
1.0	0.0821	0.0907	0.156	0.825	0.871,-0.286,0.400
1.41	0.0792	0.0877	0.140	1.6	0.854,-0.304,0.423
2.0	0.0737	0.0821	0.128	3.2	0.842,-0.324,0.432

Table captions

The first column gives values of the gluon mass in GeV . In the second and third columns the corresponding calculated high-colour pomeron intercepts per gluon are presented for $N = 2$ and $N = 3$ respectively. In the fourth column the intercepts per gluon of the multipomeronic state, taken from [1], are shown for comparison. The fifth column shows the optimal values of the variational parameter α_1 in GeV^2 . In the last column the optimal values for c_k $k = 1, \dots, N$, are presented for $N = 3$. The QCD parameter Λ has been taken to be $0.2 GeV$ for three flavours ($b = 9/4$). The calculational error for the intercept is estimated to be of the order $\pm 2 \cdot 10^{-3}$ for all values of m except $m = 0.28 GeV$ when it may rise up to $\pm 4 \cdot 10^{-3}$.